An Analysis of Space-Time codes/channel codes combination

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Abstract

In wireless communications are used several techniques to improve the reliability of transmitted data. Space-time coding technique (as Alamouti scheme) is one of them. This technique does not necessarily lead to acceptable performance in terms of Bit Error Rate and have to be combined with channel coding. Bit-interleaved coded modulation (BICM), another technique used in this field, uses a bandwidth efficient coded modulation scheme and has been shown to achieve large coding gain over fading channels, and thus widely accepted to current WLAN, DVB-S2, DSL and WiMax standards. The decoding of BICM is very similar to serially-concatenated turbo-codes and using turbo-like decoding is an appropriate iterative solution to a distributed optimization problem. Motivated by the innovation of the use of turbo codes, recently BICM with iterative decoding (BICM-ID) has been considered as an attractive approach due to its significant performance improvement with relatively low decoding complexity. The scope of this project is present a comprehensive study on the design and analysis an efficient combination of BICM transmitter scheme and of space-time codes (Alamouti scheme).

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Glossary

List of Abbreviations

APP	a posteriori probability
AWGN	additive white Gaussian noise
BCJR MAP	algorithm by Bahl, Cocke, Jelinek, Raviv
BER	bit error rate
BICM	bit-interleaved coded modulation
BICM-ID	bit-interleaved coded modulation with iterative demapping and decoding
BI-STCM-ID	bit-interleaved space-time coded modulation with iterative demapping and decoding
BPSK	binary phase shift keying
FEC	forward error correction
GSM	global system for mobile communications
HSDPA	high speed downlink packet access
i.i.d.	independent and identically distributed
LLR	log likelihood ratio
MAP	maximum a posteriori probability
MI	mutual information
MIMO	multiple-input / multiple-output
ML	maximum likelihood
OFDM	orthogonal frequency division multiplex
pdf	probability density function
PDL	parallel decoding of MLC
PSK	phase shift keying
QAM	quadrature amplitude modulation
SISO	soft-in / soft-out
SNR	signal-to-noise ratio
SOVA	soft-output Viterbi algorithm
TCM	trellis coded modulation
UMTS	universal mobile telecommunication system
WiMAX	worldwide interoperability for microwave access
WLAN	wireless local area network

Mathematical Notation

$(\cdots)^*$	conjugate of the argument
$(\cdots)^{-1}$	inverse of the argument
$(\cdots)^T$	transpose of a vector or matrix
$\lvert\cdots\rvert$	absolute value or cardinality of the argument
$\arg$	operator that delivers the argument
C	field of the complex numbers
$E\{\cdots\}$	expectation operator
$erfc(\cdots)$	complementary error function
$h(X)$	differential entropy of the random variable X
$H(X)$	entropy of the random variable X
$I(X,Y)$	mutual information between the random variables X and Y
$\mathrm{Im}(\cdots)$	imaginary part of argument
$\log(\cdots)$	natural logarithm (to base e)
$\log_2(\cdots)$	binary logarithm (to base 2)
$\log_{10}(\cdots)$	decimal logarithm (to base 10)
$\max(\cdots)$	maximum of arguments
$\min(\cdots)$	minimum of arguments
$N(\mu,\sigma^2)$	Gaussian pdf with mean μ and variance σ^2
$\mathrm{p(X)}$	probability density function of the random variable X, same as $\mathrm{p_x(X)}$
$\mathrm{P(X)}$	probability mass function of the random variable X, same as $\mathrm{p_x(X)}$
R	field of the real numbers
$\mathrm{Re}(\cdots)$	real part of argument
$\tanh(\cdots)$	hyperbolic tangent
$\mathrm{Var}(\cdots)$	variance of random variable

Chapter 1

1. INTRODUCTION:

Research interest in the field of wireless communications has grown in recent years, and this trend is very likely to continue well into the future. The wireless communication channel is the source of various weakeners to a digital communication system, due to factors such as the relative mobility of transmitter and receiver, multipath propagation, interference from other users of the frequency spectrum and fading. Several techniques are used in wireless communication to improve the reliability of transmitted data.

One of them, a space-time coding technique (as Alamouti scheme [1]), does not necessarily lead to acceptable performance in terms of Bit Error Rate and have to be combined with channel coding. Bit-interleaved coded modulation (BICM) [2], another technique used in this field, uses a bandwidth efficient coded modulation scheme and has been shown to achieve large coding gain over fading channels, and thus widely accepted to current WLAN, DVB-S2, DSL and WiMax standards. The decoding of BICM is very similar to serially-concatenated turbo-codes and using turbo-like decoding is an appropriate iterative solution to a distributed optimization problem. Motivated by the innovation of the use of turbo codes, recently, BICM with iterative decoding (BICM-ID) [3] has been considered as an attractive approach due to its significant performance improvement with relatively low decoding complexity. The scope of this project is present a comprehensive study on the design and analysis an efficient combination of BICM transmitter scheme and of space-time codes (Alamouti scheme).

Here, the comparison between the use of the different techniques are done in terms of decoding complexity, the band-width efficiency, the coding gain and the frame length by using the study tools such as BER curve characteristics and EXtrinsic Information Transfer Charts (EXIT charts) [8].

1.1. Motivation

The radio spectrum available is a limited resource and extremely costly. Therefore, an interesting view about how could be exploited the bandwidth to efficient accommodate the ever-increasing traffic demands. The different ways to transmit the sequences and different modulations and codifications are capable to achieving the substantial coding gain by expanding the multi-point in the symbol mapping keeping the bandwidth the same. The fundamental objective of the project is to study the different schemes and trying to evaluate the performance in terms of decoding complexity, the coding gain and the frame length for all of the schemes through the support of the BER characteristics and EXIT chart techniques.

2.1. Chapter organisation

This project is organized as follows:

Chapter 2 gives a brief introduction about the general framework and tools that will be used throughout the project. Then, it is defined the discrete time channel model. Moreover, it is defined Logarithmic Likelihood Ratio (LLR) and the soft metric associate to each codified bit using that definition. Lastly, the BCJR algorithm are also highlighted.

Chapter 3 presents four different transmit and received schemes used in wireless communications, such as space-time code (Alamouti), BICM and BICM-ID and an mixed scheme between Alamouti and BICM-ID scheme. Each of these different schemes studied separately in terms of signal labelling types both Gray and Set-Partitioning, interleaving and mainly the BCJR Log based decoding philosophy.

Chapter 4 investigates the EXIT chart proposed in [8] as a tool to analyse and optimize iterative receivers. Specifically, EXIT functions of the decoder and the demapper for the

BICM-ID systems are considered. A bit-level and symbol-level analysis is introduced. Therefore, a modified BI-STMC-ID scheme is detailed in this section.

Chapter 5 is dedicated to present the results of the Matlab simulations using the study tools such as the EXIT charts and BER curves for each different scheme.

Chapter 6 summarizes the results and states possible directions for future research.

Chapter 2

2. FUNDAMENTALS

2.1. Channel model

The channel model used in this work that describes the relation between the transmitted time discrete samples and the received samples . We consider only frequency non-selective channels where the channel is memoryless: the received signal depends only on the values of the transmitted signal the channel coefficient and the noise at the same time. Then we have:

The channel model is adapted such that and is the variance . Then, both the real and the imaginary part of have zero mean and variance —.

The signal-to-noise ratio (SNR) at the receiver is defined as — in dB. In order to obtain the AWGN channel model the fading coefficients are normalized to . In the other hand, to design communication systems, block fading is often considered for convenience, where changes independently after each transmitted data block. So, the fading coefficients are complex Gaussian distributed and both the real and the imaginary part of have zero mean and variance 1/2.

In both cases, the channel probability is given by the Gaussian distribution:

$$p(y|x,h) = \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{\frac{-|y-h\cdot x|^2}{2\sigma_n^2}}$$

2.2. Decision rules

For demapping and decoding the information that was codified and transmitted through channel, is necessary to compute the soft metric associate to each codified bit. In order to calculate this metrics, it is used Logarithmic Likelihood Ratio (LLR) which can be used to carry out a hard decision with respect to the transmitted bit, or a soft decision of such form to decode, for example, in iterative form.

LLRs is defined as:

$$L(d_i) = \ log \frac{P(d_i = 0)}{P(d_i = 1)}$$

where de sign of $\mathrm{L(d_i)}$ corresponds to its hard binary decision. If the value of the LLR associated to a certain bit is negative, it is possible to be supposed that it is more probable that the transmitted bit has been a 1, whereas if LLRs is positive, can be supposed that the transmitted bit was a 0, and $|\mathrm{L(d_i)}|$ corresponds a measure for reliability of the hard decision.

Therefore, to compute the soft metrics for a given channel observation *y* is used LLR:

$$L(\widehat{d_\iota}) = L(d_i|y) = \log \frac{P(d_i = 0|\ y)}{P(d_i = 1|\ y)} \ = \ \log \frac{\sum_{\forall d:\, d_i=0} P(y|\ d)P(d)}{\sum_{\forall d:\, d_i=1} P(y|\ d)P(d)}$$

2.3. Demapper:

The demapper uses the received value $\mathrm{y_n}$ of the transmitted symbol $\mathrm{x_n}$ from a 2^{m}-ary signal constellation to obtain estimates about the corresponding bits $\mathrm{d_n^m}$, m = 1,..,M. The LLR equation for estimate $\mathrm{d_n^m}$ is:

$$L(\widehat{d_{\mathrm{n}}^{m}}) = \log\left(\frac{\sum_{\forall d:\, d_{\mathrm{n}}^{m}=0} p(\mathrm{y_n} |\, \mathrm{x_n}) \prod_{j=1}^{M} P(\, d_{\mathrm{n}}^{j})}{\sum_{\forall d:\, d_{\mathrm{n}}^{m}=1} p(\mathrm{y_n} |\, \mathrm{x_n}) \prod_{j=1}^{M} P(\, d_{\mathrm{n}}^{j})}\right) =$$

$$= \log\frac{\sum_{\forall d:\, d_{\mathrm{n}}^{m}=0} p(\mathrm{y_n} |\, \mathrm{x_n}) \prod_{j=1:j\neq m}^{M} P(\, d_{\mathrm{n}}^{j})}{\sum_{\forall d:\, d_{\mathrm{n}}^{m}=1} p(\mathrm{y_n} |\, \mathrm{x_n}) \prod_{j=1:j\neq m}^{M} P(\, d_{\mathrm{n}}^{j})} + \log\frac{P(d_{\mathrm{n}}^{m}=0)}{P(d_{\mathrm{n}}^{m}=1)} =$$

$$= L_e(d_{\mathrm{n}}^{m}) + L_a(d_{\mathrm{n}}^{m})$$

where $L_e(d_{\mathrm{n}}^{m})$ corresponds to the extrinsic information and $L_a(d_{\mathrm{n}}^{m})$ corresponds to a priori information.

2.4. Decoding:

For de decoding, it is used a similar procedure than in section 2.3. The LLR equation for estimate the bits of the sequence transmitted $\widehat{\mathrm{u}_{\mathrm{t}}}$ is:

$$L(\widehat{\mathrm{u}_t}) = \log\left(\frac{\sum_{\forall u:\, \mathrm{u}_n=0} \prod_{n=1}^{N} p(\mathrm{y_n} |\, \mathrm{x_n}) \prod_{j=1}^{K} P(\, \mathrm{u}_j)}{\sum_{\forall u:\, \mathrm{u}_n=1} \prod_{n=1}^{N} p(\mathrm{y_n} |\, \mathrm{x_n}) \prod_{j=1}^{K} P(\, \mathrm{u}_j)}\right) =$$

$$= \log\frac{\sum_{\forall d:\, d_{\mathrm{n}}^{m}=0} p(\mathrm{y_n} |\, \mathrm{x_n}) \prod_{j=1:j\neq m}^{M} P(\, d_{\mathrm{n}}^{j})}{\sum_{\forall d:\, d_{\mathrm{n}}^{m}=1} p(\mathrm{y_n} |\, \mathrm{x_n}) \prod_{j=1:j\neq m}^{M} P(\, d_{\mathrm{n}}^{j})} + \log\frac{P(\mathrm{u}_n=0)}{P(\mathrm{u}_n=1)} =$$

$$= L_e(\mathrm{u}_n) + L_c(\mathrm{u}_n) + L_a(\mathrm{u}_n)$$

where $L_e(\mathrm{u}_n)$ corresponds to the extrinsic information, $L_c(\mathrm{u}_n)$ corresponds to the channel information and $L_a(\mathrm{u}_n)$ corresponds to a priori information.

2.5. BCJR Algorithm

The BCJR algorithm was developed by L.Bahi, J.Coocke, F Jelinek and J.Raviv in 1974. The algorithm is suitable for estimating bit and symbol probabilities for a finite-state Markov source transmitter through a discrete memoryless channel. The further details of the BCJR algorithm can be seen in [7].

As it's explained in section 2.2, the soft information can be expressed by Logarithmic Likelihood Ratios (LLR's).

The Log based BCJR algorithm is selected due to has many advantages. One of them is that since the calculation is done in the log domain it can avoid unnecessary numerical overflows. Therefore, multiplication and division are defined as addition and substation by the properties of the logarithm.

So, the aim of the Log-BCJR algorithm is to calculate the extrinsic LLR from the corresponding decoded sequence. The calculation of extrinsic LLR m_t lead to the calculation of the three internal variables: γ, α and β.

(i) The γ (t) value represents the conditional probability that corresponds to each transition in trellis. That is the probability of a transition between the states $\mathrm{S_r}$ and $\mathrm{S_s}$ based on what we know about y_t.

(ii) The $\alpha(S_r)$ values are the forward recursion of the BCJR decoder and it corresponds to the probability that the encoder is in state $\mathrm{S_r}$ at time t-1 based on what we know about y_t^-.

(iii) The $\beta(S_s)$ values are the backward recursion of the BCJR decoder and it corresponds to the probability that the encoder is in state $\mathrm{S_s}$ at time t based on what we know about y_t^+.

where $\mathrm{y_t^+}$ represents the values received for the set of bits sent after time t and $\mathrm{y_t^-}$ the values received for the set of bits sent before time t, and $\mathrm{S_r}$ and $\mathrm{S_s}$ the values of the state at times $\mathrm{t-1}$ and time t respectively.

So,

$$\alpha_{t-1}(S_r) = p\ (S_r, \mathrm{y_t^-})$$

$$\beta_t(S_s) = p\ (\mathrm{y_t^+} |\ S_s)$$

$$\gamma_t(Sr\ , Ss\ ) = p\ (\ S_s, y_t|S_r)$$

$$m_t(Sr, Ss\ ) =\ p\ (Sr, Ss\ , y)$$

Then,

$$m_t(Sr\ , Ss\ ) = \alpha_{t-1}(S_r)\ \gamma_t(Sr, Ss\ )\ \beta_t(S_s)$$

We will use the capital letters A, B and Γ for the log metrics of α, β and γ and write, for states S_r and S_s ,

$$\Gamma_{\mathrm{t}}(S_r, S_s) = \log \gamma_t\ (S_r, S_s) = \log p(\mathbf{u}_t\ =\ \mathbf{u}_{r,s})\ p(\mathbf{y}_t\,|\mathbf{c}_{r,s})$$

$$=\ \log p(\mathbf{u}_t\ =\ \mathbf{u}_{r,s})\ +\ \log p(\mathbf{y}_t\,|\mathbf{c}_{r,s})$$

$$A_t\ (S_r) = \log \alpha(S_r) = \ \log \sum_{\mathrm{i}} \alpha_{t-1}(S_i)\ \gamma_t(S_i, S_r) = \ \log\ \sum_{\mathrm{i}} e^{A_{t-1}(S_i)+\Gamma(S_i,S_r)}$$

$$B_t\ (S_s) = \log \beta(S_s) = \ \log \sum_{\mathrm{i}} \beta_{t+1}(S_i)\ \gamma_{t+1}(S_s, S_i) = \ \log\ \sum_{\mathrm{i}} e^{B_{t+1}(S_i)+\Gamma_{t+1}(S_s,S_i)}$$

$$M_t(Sr\ ,Ss\ ) = log\ m_t(Sr\ ,Ss\ ) = A_{t-1}\ (S_r) +\ \Gamma_{\mathrm{t}}(S_r\ ,S_s\ ) + B_t\ (S_s)$$

where $\mathrm{p(a)} = e^{\log \mathrm{p(a)}}$.

Finally, the extrinsic LLRs of the uncoded bits are calculated with aid of Jacobian logarithm.

The Jacobian logarithm can be defined as:

$$\begin{aligned} f(\phi_1, \phi_2) &= \ln\left(e^{\phi_1} +\ e^{\phi_2}\right) \\ &= max\{\phi_1, \phi_2\} + \ln\left(1 + e^{-|\phi_1 - \phi_2|}\right) \\ &= max^*\{\phi_1, \phi_2\} \end{aligned}$$

where $\ln\left(1 + e^{-|\phi_1 - \phi_2|}\right)$ is implemented by the a aid of lookup table. This version of Log Bases BCJR which is realised by looking the lookup table is known as Approx-Log-Bases BCJR algorithm.

Using LLRs as input and output, the pseudo code for the log BCJR decoding of a binary convolutional code is given in Appendix 1. The inputs are the a priori message bit LLRs A, the received LLRs R and the length-T convolutional trellis with 2^{v} states. The Γ, A and B values are calculated using the previous equations. The outputs are the APPs of the kT message bits calculated using the equations provided in section 2.3 and 2.4.

2.6.Viterbi Algorithm

Viterbi Algorithm was developed by Andrew Viterbi [], it is established that the algorithm calculates the maximum likelihood code sequence from the received data. It is applied, majority in application such as GSM phones, space probes etc. The Virtebi algorithm can be done either by hard decoding or soft decoding.

a. Virterbi Hard Decoding

The principle of Virterbi, the algorithm operates from the zero state (supposing a 2 bit symbol), and after that, it will compare the output of the received signal with respect to the encoded sequence of the trellis, through the basis of the Hamming distance. During this stage, decoder is unable to express any preference to the whether it was 00 or 11 was more. These Hamming distance are known as the context of the Viterbi decoding and known to be the branch metric. Now proceeding to next symbol it will again compute the hamming distance of all possible four legitimate paths and the received signal. This distance will yield to the new branch metric associated with second trellis stage. By now the encoded symbol of two original input bits have been reside. Now the obtained branch metric is added to previous branch metric to obtain the path metric . A low Hamming distance can indicates a high similarity between the received sequence and the encoded sequence concerned, which is characteristic of the most likely encoded sequence, since the probability of a high number of error is exponentially decreasing with numbers of error.

b. Virterbi Soft Decoding

In the hard decision, Viterbi decoding, based on the location of the received coded symbol, the coded bit was estimated if the received symbol is greater than zero, the received coded bit is 1; if the received symbol is less than or equal to zero, the received coded bit is 0.

But in Soft decision decoding, rather than estimating the coded bit and finding the Hamming distance, the distance between the received symbol and the probable transmitted symbol is found out. This is done as more levels of confidence. For example, in eight symbols mapping, scale +4 indicates the highest possible confidence concurring the demodulator's decision for a binary 1 and -4 for the lowest possible confidence. In fact, if the demodulator output -4 , the low confidence of in a logical implies a high probability of a binary zero. So, thanks to this eight level confidence scale, the received bits can be decoded with lower error probability.

Chapter 3

3. THE PROPOSED SCHEMES

In this section three different schemes are provided. It is described the system model of space–time code (Alamouti). And also the models BICM and BICM-ID are described.

It is also analysed and designed an efficient combination of BICM scheme and space-time codes (Alamouti scheme) in order to find an optimal value of the variable   and to try to improve the performance of the system.

3.1. STBC scheme (Alamouti Scheme)

Most of dispersive environments, the antenna diversity is a practical and effective technique and, for these reason, widely applied to reduce fading multi-way effect. The classic solution is to use multiple antennas in the receiver with some methods of combination to improve the quality of the received signal. But when using diversity in reception in movable systems is the cost, size and power consumption in the movable units. The use of multiple antennas and chains of radio frequency (or circuits of selection and commutation) causes that the movable units are great and expensive. However a station base often serves to hundreds or thousands as movable units, and is therefore economic to add equipment to the stations it bases before to the movable units. For these reasons, the diversity techniques have been almost exclusively applied to the stations bases.

Based on these techniques, Alamouti presents a scheme of diversity in transmission of open bow using two transmitting antennas and one receiving antenna that easily extends to the use of several receiving antennas; scheme that later would know like code space-time blocks (STBC). The STBC provides complete space diversity and makes use of a very simple algorithm of decoding that it only requires of linear processing on the received signals.

Using two transmitting antennas and one antenna in reception the scheme obtained the same order of diversity that scheme MRRC (maximal-ratio to receiver combining: combined receiver of maximum rate) with a transmitting antenna and two receiving antennas.

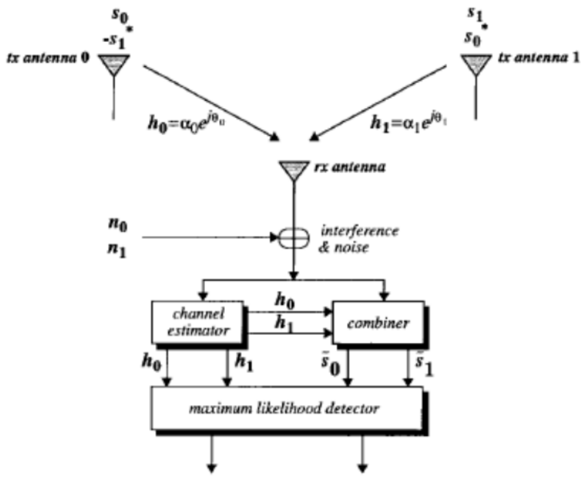


Figure 01.- Two transmitting and a simple receiving antenna scheme

a. Transmitted sequence:

Each period symbol, both antennas transmit simultaneously a signal. The signal sent by antenna 0 is denominated s_0, whereas the transmitted one by antenna 1 is s_1. In the following period of symbol, antenna 0 transmits the signal $-s_1{}^*$, and the signal $s_0{}^*$ is the signal that will send antenna 1.

	Antenna 0	Antenna 1
time *t*	s_0	s_1
time *t* + *T*	$-s_1{}^*$	$s_0{}^*$

Table 01.- Alamouti signal space-time codification scheme

b. Channel model

At the time t, the channel can be expressed by a complex multiplicative distortion $\mathrm{h}_0(\mathrm{t})$ for the transmit antenna zero and $\mathrm{h}_1(\mathrm{t})$ transmitting antenna one. Then, assuming that the fading is constant throughout two consecutive symbols, the channel is expressed by:

$$h_0(t) = h_0(t+T) = h_0 = \alpha_0 e^{j\theta_0}$$

$$h_1(t) = h_1(t+T) = h_1 = \alpha_1 e^{j\theta_1}$$

c. Receiver

At the receiver, the signals obtained r_0 and r_1 are combined and sent it to the maximum likelihood detector which, for each of the combined signals s_0 and s_1, uses the rule decision rule expressed by:

At time t:

$r_0 = h_0 s_0 + h_1 s_1 + n_0$ -------> $\hat{s}_0 = {h_0}^* r_0 + h_1 {r_1}^*$

At time t+1:

$r_1 = -h_0 {s_1}^* + h_1 {s_0}^* + n_0$ -------> $\hat{s}_1 = {h_1}^* r_0 - h_0 {r_1}^*$

Using the decision rule:

$$\mathrm{d}^2(\hat{\mathrm{s}}_0, \mathrm{s}_\mathrm{i}) \leq \mathrm{d}^2(\hat{\mathrm{s}}_0, \mathrm{s}_\mathrm{k})$$

where $\mathrm{d}^2(\mathrm{x}, \mathrm{y})$ represents the squared Euclidean distance between signals x and y.

There exist applications where an order of greater diversity is required and can be used multiple antennas in the side of the receiver. In these cases, it is possible to obtain an order of diversity of 2M with two transmitters and M antennas in reception. The case of two antennas in more detail will be explained here. The following figure (Fig. 02) presents a possible scheme for the case of two receiving antennas:

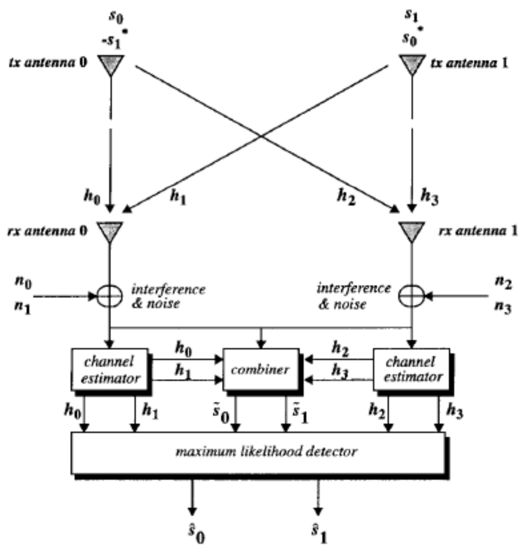


Figure 02.- Two transmitting and a simple receiving antenna scheme

Using two antennas at the transmitter, the scheme doubles the diversity order of systems with one transmit antenna and multiple receive antennas [1].

3.2. BICM (Bit Interleaved Coded Modulation)

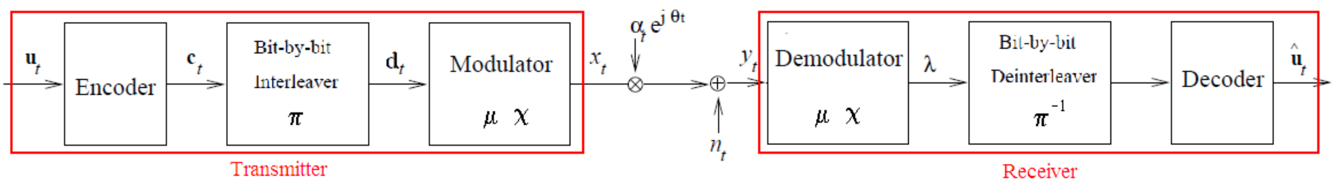


Figure 03.- Bit-interleaved coded Modulation (BICM) scheme

Bit-interleaved Coded Modulation (BICM) was the idea proposed by Zehavi [4] in order to improve the diversity of the code in Rayleigh channel. The design of the coded modulation schemes are affected by several factors such as High Free Euclidian Distance which is desired for the AWGN channel, while its was interested to note that a high Effective Code Length and a high minimum product distance were the main factors effecting the fading channel. The diversity of the code can be defined as ”length” of the shortest error path and one should be aware that the shortest error distance are not necessarily be the minimum distance error.

As much BICM as BICM-ID shares the same scheme of transmitter. However, the receiver is different due to BICM-ID implement an iterative decoding.

As is shown in Fig. 03, the transmitter block diagram of BICM is developed by a serial concatenation of convolutional encoder, a random bit-by-bit interleaver (π), and a memoryless modulator $(\mu[\cdot])$.

The information block bits u_t is codified with a rate of code R, defined as R= K/N. Denote the encoder input bit at time *t* by $u_t = [u_t^1, \dots, u_t^i, \dots, u_t^K]$ and the output bit by $c_t = [c_t^1, \dots, c_t^i, \dots, c_t^N]$ where u_t^i or c_t^i is the *i*-th bit in the sequence. The encoder output is bitwise interleaved $d_t = [d_t^1, \dots, d_t^i, \dots, d_t^N]$. The main purpose of the bit interleaver is to break the sequential fading correlation and increase the diversity order to the minimum Hamming distance of the convolutional code.

Each *m* consecutive bits of the interleaved sequence are grouped to form a channel symbol in the modulator map $s_t = \mu[d_t]$ chosen from M-ary constellation $s_t \in \chi = \{\alpha_1, \alpha_2, \dots, \alpha_M\}$, where m is the labelling map and M = 2^m.

The constellations considered in this work are M-QAM with M ∈ {4, 8, 16, 32, 64} or M-PSK with M ∈ {8, 16, 32} which uses a Gray mapping or Set Partitioning mapping:

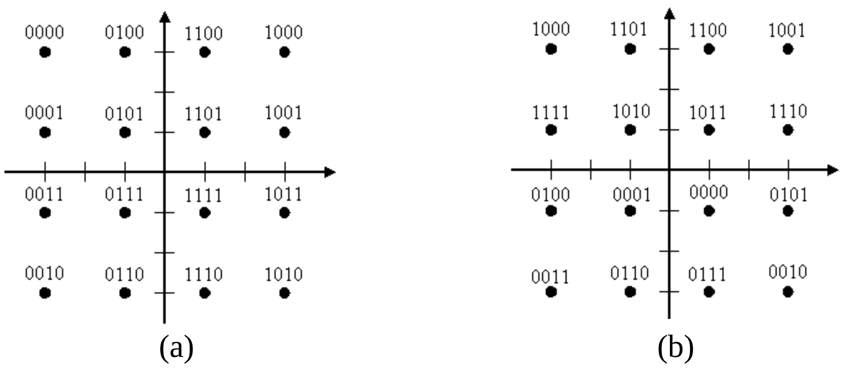


Figure 04.- 16 QAM signal set with (a) Gray labelling and (b) set partitioning.

All these constellations they are standardized in power and they own null average. That is to say:

$$\sum_{i=1}^{M} \alpha_i = 0 \qquad \frac{1}{M}\sum_{i=1}^{M} |\alpha_i|^2 = 1$$

The discrete-time complex baseband received signal can be written as:

$$y_t = x_t * h_t + n_t$$

where h_t is the complex fading coefficient, and n_t is the complex white Gaussian noise sample with null average and variance $N_0 = \{|n_t|^2\}$.

By assuming that the receiver has perfect channel state information (CSI), the demapping is defined using LLRs (section 2.2) as suboptimal maximum log-likelihood bit metrics are obtained as:

$$\lambda\left(d_t^i = b\right) = log\, P\left(d_t^i = b\right) \cong -min_{s_t \epsilon \chi_b^i} \left|y_t - \alpha_t e^{j\theta_0} \cdot x_t\right|^2$$

The branch metrics are obtained by summing the corresponding deinterleaved bit metrics before being passed to the Viterbi decoder.

3.3. BICM-ID (Bit Interleaved Coded Modulation - Iterative Decoding)

Bit-Interleaved Modulation was purposed to increase the diversity of the Ungerboeck TCM scheme under the Rayleigh channel. In [9] is suggested a new scheme of Bit-Interleaved Coded Modulation using Iterative Decoding which employed Set-Partitioning signal labelling system as that of Ungerboeck. Introduction of soft-decision feedback from the decoder’s output to the demapper/demodulator input to iterate between them is advantageous of the fact that, it improves the reliability of the soft information passed to the demapper/demodulator.

So, in BICM with iterative demapping and decoding (BICM-ID) scheme it is used the same transmitter than in BICM scheme, explained in the section 3.2. On the other hand, BICM-ID’s receiver is almost similar to that of BICM’s receiver except the fact that the iterative process is used in order to achieve global optimum through a step-by step local search. Iterative decoding is recent success in Forward error correcting codes and can achieve a rate equivalent to that Shannon capacity.
The scheme of BICM-ID’s receiver is shown in Fig. 05.

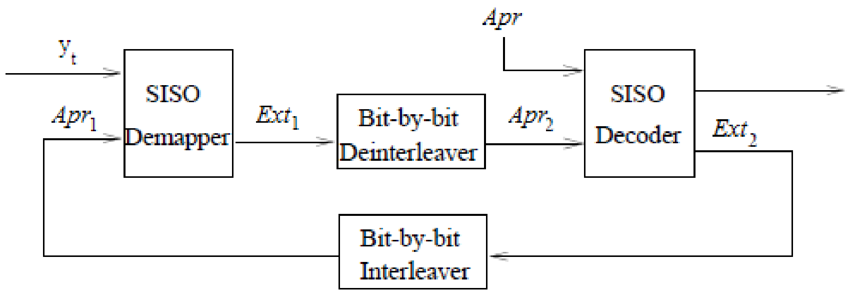


Figure 05.- BICM-ID's receiver

The first block of this receiver is the demapper block. This block computes the received complex signals $\mathrm{y_t}$ and output the LLRs, $L\left(\widehat{d_\mathrm{n}^m}\right)$ m = 1, ... , M of the corresponding coded bits.

Using the procedure shown in 2.3 and 2.4, and defining $\mathrm{Ext_1} = \mathrm{L_e(d_n^m)}$, corresponds to the extrinsic information and $\mathrm{Apr_1} = L_a(d_\mathrm{n}^m)$ corresponds to a priori information:

Demapping:

$$L\left(\widehat{d_\mathrm{n}^m}\right) = L_e(d_\mathrm{n}^m) + L_a(d_\mathrm{n}^m)$$

$$Ext_1 = L_e(d_\mathrm{n}^m) = L\left(\widehat{d_\mathrm{n}^m}\right) - L_a(d_\mathrm{n}^m)$$

$$Apr_2 = \Pi(Ext_1)$$

With BICM-ID scheme, large gains over the iterations are achieved in practical systems with carefully chosen mappings different from Gray. In this case, it is used set partitioning labelling (Fig. 03-b) which is better suited for BICM-ID than Gray labelling (Fig. 03-a) [3].

Decoding:

$$L(\widehat{\mathrm{u}_t}) = L_e(\mathrm{u}_n) + L_c(\mathrm{u}_n) + L_a(\mathrm{u}_n)$$

$$Ext_2 = L_e(\mathrm{u}_n) = L(\widehat{\mathrm{u}_t}) - L_c(\mathrm{u}_n) - L_a(\mathrm{u}_n)$$

$$Apr_1 = \Pi^{-1}(Ext_2)$$

Therefore, as it is shown in Fig. 05, the *Ext1* is interleaved and used as an input of the SISO decoder.

In the SISO decoder block, the BCJR algorithm [7, Appendix A] is used. Then, the channel decoder uses this BCJR algorithm to compute extrinsic estimates (*Ext2*) about the coded bits that are feedback and regarded as a priori information (*Apr1*) at the demapper.

As it is shown in Fig. 05 also, the SISO demapper uses a priori information outputs from the decoder (Apr1) to achieve global optimum through a step-by-step local search. Iterative decoding is recent success in Forward error correcting (FEC) codes and can achieve a rate equivalent to that Shannon capacity.

3.4.STBC & BICM-ID

One of the most important future advances in wireless communications will be the high-data-rate applications, particularly for systems that are power, bandwidth, and complexity limited. Motivated by the need, the use of multiple transmit and receive antennas is extensively exploited to significantly increase channel capacity. Pioneering works predict notable spectral efficiencies for multiple-input multiple-output (MIMO) systems. So, the use of MIMO schemes also provides higher diversity order that can be exploited to combat severe attenuation in a multipath wireless environment. For this reason, the designs of space-time codes for achieving these advantages, combining, at the same time, with bit-interleave coded modulation was proposed as an effective approach to capture both space and time diversity.

In [10], bit-interleaved space-time coded modulation (BI-STCM) (without iterative decoding) was proposed as an effective approach to capture both space and time diversity, but no tight bound on bit error rate (BER) was given.

In this chapter, it is considered BI-STCM with iterative decoding (BI-STCM-ID) over Rayleigh fading MIMO channels when perfect CSI is available to the receiver.

The scheme of the system model of BI-STCM-ID with N_T transmit and N_R receive antennas is shown in Fig. 06 and Fig. 07.

a. BI-STCM-ID Transmitter

The BI-STCM-ID transmitter scheme Fig. 6 is a serial concatenation of the conventional BICM and a space-time block code (STBC). So, as in the BICM transmitter, the information sequence is first encoded by a convolutional code of rate R=K/N. The encode output is bit-interleaved and, after that, mapped choosing a 2^m-ary constellation according to the labelling map μ. Then, the consecutive modulated symbols (the output's modulator) are the inputs of an STBC block, which will be transmitted at the k-th time interval consisting of MIMO channel. That is the space-time block encoder takes the constellation signals to form a $[\,T\ x\ n_T]$ space-time codeword matrix $\boldsymbol{X_t} = G(s[k])$.

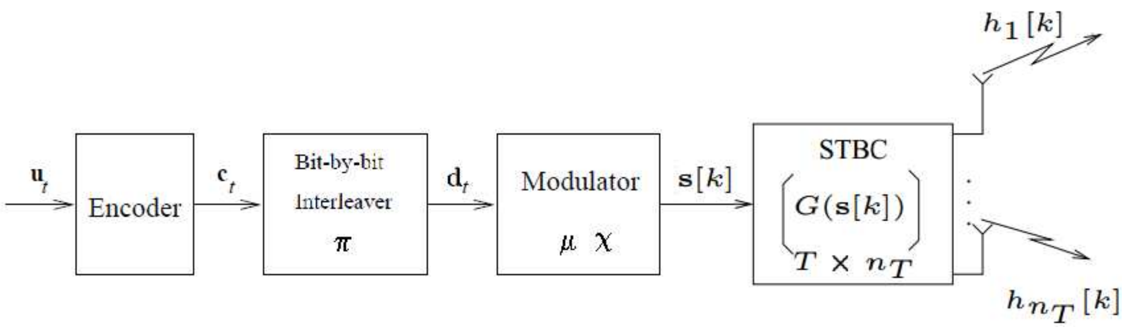


Figure 06.- BI-STCM-ID transmitter scheme

b. Channel model

As it is mentioned above, N_r receive antennas capture the signals transmitted by the N_t antennas from the transmitter. Consequently, the channel is described by a $N_t\,x\,N_r$ matrix H_k

$$y_k^r = \sqrt{E_s}\sum_{t=1}^{N_t} x_k^r h_k^{r,t} + n_k^r$$

where $h_k^{r,t}$ is defined as the equivalent channel response between the t-th transmit antenna and the r-th receive antenna, at time kT. And n_k^r is a sequence of i.i.d. complex Gaussian variables with zero mean and variance $\frac{N_0}{2}$.

So, the final vector notation of signal received y_k^r can be written as:

$$\underline{y_k} = \begin{bmatrix} y_k^1 \\ \cdots \\ y_k^{N_r} \end{bmatrix} = \begin{bmatrix} h_k^{1,1} & \cdots & h_k^{1,N_t} \\ \vdots & \ddots & \vdots \\ h_k^{1,N_r} & \cdots & h_k^{N_r,N_t} \end{bmatrix} \begin{bmatrix} x_k^1 \\ \cdots \\ x_k^{N_t} \end{bmatrix} + \begin{bmatrix} n_k^1 \\ \cdots \\ n_k^{N_r} \end{bmatrix}$$

$$\underline{y_t} = \underline{H_t} \cdot \underline{x_t} + \underline{n_t}$$

c. BI-STCM-ID iterative Decoder

In Fig. 07 the BI-STCM-ID receiver scheme is depicted. Consider the channel state information (CSI) is perfectly known at the receiver.

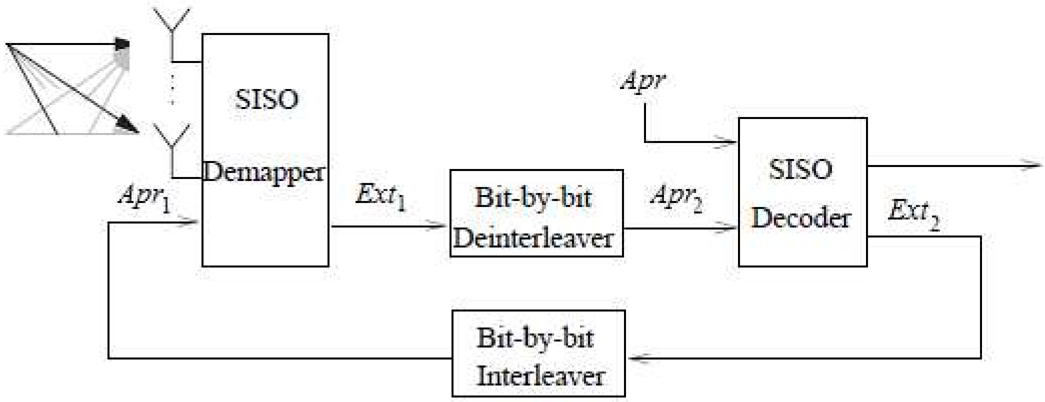


Figure 07.- BI-STCM-ID receiver scheme

The demapper block uses the maximum a posteriori (MAP) decoding algorithm and the metric associated with each received signal y_t is the log pdf:

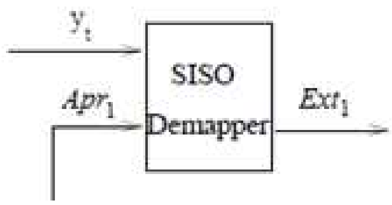
$$\Lambda(\underline{y_t}|\underline{x_t}) = \log p(\underline{y_t}|\underline{x_t}) = -\frac{1}{N_0} \left\| \underline{y_t} - \underline{H_t} \cdot \underline{x_t} \right\|^2$$

and LLR for the unmapped bit of Apr2 is used as it is explain in section 2.3.

The SISO decoder takes Apr2 (bit-deinterleaved of Ext1) to compute the extrinsic LLR of each coded bit, which is fed back to the demapper as the updated a priori information Apr1 on the next iteration.

3.5.Modified BI-STCM-ID scheme

The a priori LLR information of the outer convolution code can be modelled by an independent zero-mean Gaussian random variable and variance σ^2. Thanks to know that, in this modified BI-STCM-ID scheme, the a priori information is providing as an independent source for the iterative receive with these features.
Assuming that χ is a random variable, it is possible to



$\chi + n(0, \sigma^2)$

Figure 08.- Modified BI-STCM-ID demmapper scheme

So, instead of doing several iterations to obtain the optimal value of LLR (a priori information), in this case, we use an independent Gaussian source to pretend the iterations.
As a result, the source Ext1 (it is LLR) can be expressed as:

$$Ext_1 = \frac{\sigma'^2}{2}\chi + n(0, \sigma'^2)$$

To emulate the iterative system, it is added a multiplying parameter $\beta \in [0, 1]$, that is considered in this modified scheme.

$$Ext_1 = \beta \cdot \frac{\sigma'^2}{2}\chi + n(0, \sigma'^2)$$

We take as a reference when the value of β is equal to 1 ($\beta = 1$)
And since the χ is a random variable, but known beforehand, it is possible to calculate de β value computing the next equation:

$$\chi \cdot Ext_1 = \beta \cdot \frac{\sigma'^2}{2} \chi^2 + \chi \cdot n(0, \sigma'^2)$$

Once we know the value of $(\chi \cdot Ext_1)$, it is directly to calculate its mean and its variance:

$$E[\,\chi \cdot Ext_1] = \beta \cdot \frac{\sigma'^2}{2}$$

$$Var[\,\chi \cdot Ext_1] = \sigma'^2$$

Therefore, to find the optimal value of β:

$$\frac{E[\,\chi \cdot Ext_1]}{Var[\,\chi \cdot Ext_1]} = \frac{\beta}{2} \qquad\qquad \beta = \frac{2 \cdot E[\,\chi \cdot Ext_1]}{Var[\,\chi \cdot Ext_1]}$$

Chapter 4

4. EXTRINSIC INFORMATION TRANSFER CHART

The EXIT chart was investigated as a tool to analyze the convergence properties of iterative receivers and in particular of bit-interleaved coded modulation with iterative demapping and decoding (BICM-ID).

4.1. EXIT chart analysis

The **EX**trinsic **I**nformation **T**ransfer chart (EXIT chart) is a tool, proposed in [8], used to analyse the convergence behaviour of systems that use soft iterative decoders. EXIT chart are particularly useful for analyse the behaviour at cliff region. The former visualize the exchange of extrinsic information between the constituent decoders, while the latter analyses the decoding behaviour by tracking the densities of the messages throughout the iterations. Calculating and tracking the densities can be difficult, specially, if more complicated component codes are used like the ones in our receiver design. Note that we are analysing a serially concatenated scheme over an AWGN channel with the BICM as the outer constituent decoder and the BCJR detector as the inner constituent decoder. Assuming that the bit-interleaver is sufficiently large, the EXIT charts provide an accurate prediction of the convergence behaviour.

The EXIT chart characterizes the relation between input and output LLRs of the component decoders in terms of mutual information. Shannon's mutual information between the transmitted bits and the LLR values L is used to measure the information contents of the a priori knowledge. For simplicity, let us assume binary phase-shift keying (BPSK) modulation and a large interleaver, which assures statistical independence and Gaussian distribution for the input L with parameter $\sigma^2 \leftrightarrow I(Y_1\,;Z)$. As it is depicted in Fig. 9, we use the quality of the a priori information and observe the extrinsic output information. L_A denotes the a priori input LLR and L_E refers to the extrinsic output LLR.

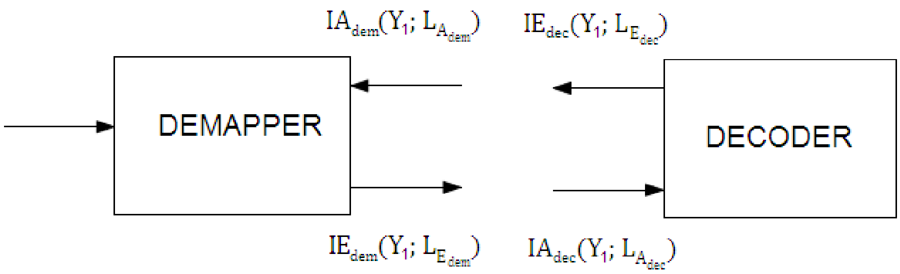


Figure 09.- Modelling a serial concatenated system with EXIT functions

Considering Y and Z to be two real valued random variables, the mutual information is defined as

$$I(Y_1;Z) = \int\int P_{L_a}(z|Y_1 = a_n)\cdot \log \frac{P_{L_a}(z|Y_1 = a_n)}{P_{L_a}(z)}\, dz\, dy$$

$$I(Y_1;Z) = \frac{1}{m}\sum_{n=1}^{m}\int_{-\infty}^{+\infty} P_{L_a}(z|Y_1 = a_n)\cdot \log \frac{P_{L_a}(z|Y_1 = a_n)}{P_{L_a}(z)}\, dz$$

where

$$P_{L_a}(z|Y_1 = a_n) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\left[\frac{(z-a_n)^2}{2\cdot\sigma^2}\right]}$$

and

$$P_{L_a}(z) = \frac{1}{N_A}\cdot\sum_{n=1}^{N_A} P_{L_a}(z|Y_1 = a_n)$$

If we restrict ourselves to $y \in \{+1, -1\}$, and equally likely inputs y, the mutual information is defined as:

$$I(Y_1;Z) = \frac{1}{2}\sum_{Y_1=-1,1}\int_{-\infty}^{+\infty} P_L(z|Y_1)\cdot \log \frac{2\cdot P_L(z|Y_1)}{P_L(z|Y_1 = -1) + P_L(z|Y_1 = +1)}\, dz$$

Therefore, the a priori mutual information I_{A_1} and extrinsic mutual information I_{E_2} is, respectively:

$$I_{A_1}(Y_1;Z) = \frac{1}{2}\sum_{Y_1=-1,1}\int_{-\infty}^{+\infty} P_{L_{A_1}}(z|Y_1)\cdot\log\frac{2\cdot P_{L_{A_1}}(z|Y_1)}{P_{L_{A_1}}(z|Y_1=-1)+P_{L_{A_1}}(z|Y_1=+1)}dz$$

$$I_{E_2}(Y_1;Z) = \frac{1}{2}\sum_{Y_1=-1,1}\int_{-\infty}^{+\infty} P_{L_{E_2}}(z|Y_1)\cdot\log\frac{2\cdot P_{L_{E_2}}(z|Y_1)}{P_{L_{E_2}}(z|Y_1=-1)+P_{L_{E_2}}(z|Y_1=+1)}dz$$

These integrals have to be evaluated numerically. So, we first define $I_{A_1}^{+1}$ and $I_{A_1}^{-1}$ as

$$\begin{aligned}
I_{A_1}^{+1}(Y_1;Z) &= \int_{-\infty}^{+\infty} P_{L_{A_1}}(z|Y_1=+1)\cdot\log\frac{2\cdot P_{L_{A_1}}(z|Y_1=+1)}{P_{L_{A_1}}(z|Y_1=-1)+P_{L_{A_1}}(z|Y_1=+1)}dz \\
&= \int_{-\infty}^{+\infty} P_{L_{A_1}}(z|Y_1=+1)\cdot\log\frac{2}{1+\dfrac{P_{L_{A_1}}(z|Y_1=-1)}{P_{L_{A_1}}(z|Y_1=+1)}}dz \\
&= E_{Y=+1}\left\{1-\log[1+\frac{P_{L_{A_1}}(z|Y_1=-1)}{P_{L_{A_1}}(z|Y_1=+1)}]\right\}
\end{aligned}$$

$$\begin{aligned}
I_{A_1}^{-1}(Y_1;Z) &= \int_{-\infty}^{+\infty} P_{L_{A_1}}(z|Y_1=-1)\cdot\log\frac{2\cdot P_{L_{A_1}}(z|Y_1=-1)}{P_{L_{A_1}}(z|Y_1=-1)+P_{L_{A_1}}(z|Y_1=+1)}dz \\
&= \int_{-\infty}^{+\infty} P_{L_{A_1}}(z|Y_1=-1)\cdot\log\frac{2}{1+\dfrac{P_{L_{A_1}}(z|Y_1=+1)}{P_{L_{A_1}}(z|Y_1=-1)}}dz \\
&= E_{Y=-1}\left\{1-\log[1+\frac{P_{L_{A_1}}(z|Y_1=+1)}{P_{L_{A_1}}(z|Y_1=-1)}]\right\}
\end{aligned}$$

to express I_{A_1} as

$$I_{A_1} = \frac{1}{2}\left(I_{A_1}^{+1} + I_{A_1}^{-1}\right)$$

$$= \frac{1}{2}\left[E_{Y=+1}\left\{1 - \log[1 + \frac{P_{L_{A_1}}(z|Y_1 = -1)}{P_{L_{A_1}}(z|Y_1 = +1)}]\right\}\right.$$

$$\left. + E_{Y=-1}\left\{1 - \log[1 + \frac{P_{L_{A_1}}(z|Y_1 = +1)}{P_{L_{A_1}}(z|Y_1 = -1)}]\right\} \right]$$

If

$$I_{A_1} = \frac{1}{2}\left\{\frac{1}{N_{Y=+1}}\sum_{n=1}^{N_{Y=+1}}\left[1 - \log[1 + \frac{P_{L_{A_1}}(z|Y_1 = +1)}{P_{L_{A_1}}(z|Y_1 = -1)}]\right]\right.$$

$$\left. + \frac{1}{N_{Y=-1}}\sum_{n=1}^{N_{Y=-1}}\left[1 - \log[1 + \frac{P_{L_{A_1}}(z|Y_1 = +1)}{P_{L_{A_1}}(z|Y_1 = -1)}]\right]\right\}$$

where $\mathrm{N}_{\mathrm{Y}=+1}$ is the number of bits 0 and $\mathrm{N}_{\mathrm{Y}=-1}$ is the number of bits 1. Therefore, we define N as the total number of bits in one simulation block $\mathrm{N} = \mathrm{N}_{\mathrm{Y}=+1} + \mathrm{N}_{\mathrm{Y}=-1}$ and with assumed equally likely $\mathrm{N}_{\mathrm{Y}=+1} = \mathrm{N}_{\mathrm{Y}=-1} = \frac{\mathrm{N}}{2}$, it is easy simplify the last equation as:

$$I_{A_1} = \frac{1}{2}\left\{\frac{1}{N/2}\sum_{n=1}^{N/2}\left[1 - \log[1 + \frac{P_{L_{A_1}}(z|Y_1 = +1)}{P_{L_{A_1}}(z|Y_1 = -1)}]\right]\right.$$

$$\left. + \frac{1}{N/2}\sum_{n=1}^{N/2}\left[1 - \log[1 + \frac{P_{L_{A_1}}(z|Y_1 = +1)}{P_{L_{A_1}}(z|Y_1 = -1)}]\right]\right\}$$

$$= 1 - \frac{1}{N}\sum_{n=1}^{N}\log(1 + e^{-d_n \cdot \lambda_{A_n}})$$

For simplify de nomenclature, $P_{L_{A_{1n}}} = P_{A_n}$

$$\lambda_{A_n} = \log \frac{P_{A_n}(z|Y_1 = +1)}{P_{A_n}(z|Y_1 = -1)}$$

In order to detect a possible error, we can be writing as $\lambda_{A_n} = sign(\lambda_{A_n}) \cdot |\lambda_{A_n}|$. So, if there is an error, we can detect it, because $d_n \cdot |\lambda_{A_n}|$ = -1. And that occurs with probability of

$$P_{A_n} = \frac{e^{+|\lambda_{An}/2|}}{e^{+|\lambda_{An}/2|} + e^{-|\lambda_{An}/2|}}$$

Then,

$$\log(1 + e^{-d_n \cdot \lambda_{A_n}}) = P_{A_n} \cdot \log(1 + e^{-d_n \cdot \lambda_{A_n}}) + (1 - P_{A_n}) \cdot \log(1 + e^{-d_n \cdot \lambda_{A_n}})$$
$$= P_{A_n} \cdot \log(P_{A_n}) + (1 - P_{A_n}) \cdot \log(1 - P_{A_n}) = H_b(P_{A_n})$$

And if is substituted this result from the previous equation of I_{A_1}, finally we can write:

$$I_{A_1} = 1 - \frac{1}{N} \sum_{n=1}^{N} H_b(P_{A_n})$$

The same way, and if it is used the same procedure, it is possible to define I_{E_2} as:

$$I_{E_2} = 1 - \frac{1}{N} \sum_{n=1}^{N} H_b(P_{E_n})$$

where

$$P_{E_n} = \frac{e^{+|\lambda_{En}/2|}}{e^{+|\lambda_{En}/2|} + e^{-|\lambda_{En}/2|}}$$

Fig. 10 shows the extrinsic iterative decoding trajectory that is connected through the interleaver, the extrinsic outputs of the demapper becomes the a priori information to the decoders and the extrinsic output of the decoder become becomes the a priori inputs to demapper. This exchange is what is known as EXIT chart and it can be plotted into a single diagram by combing both the demapper and decoder transfer characteristics.

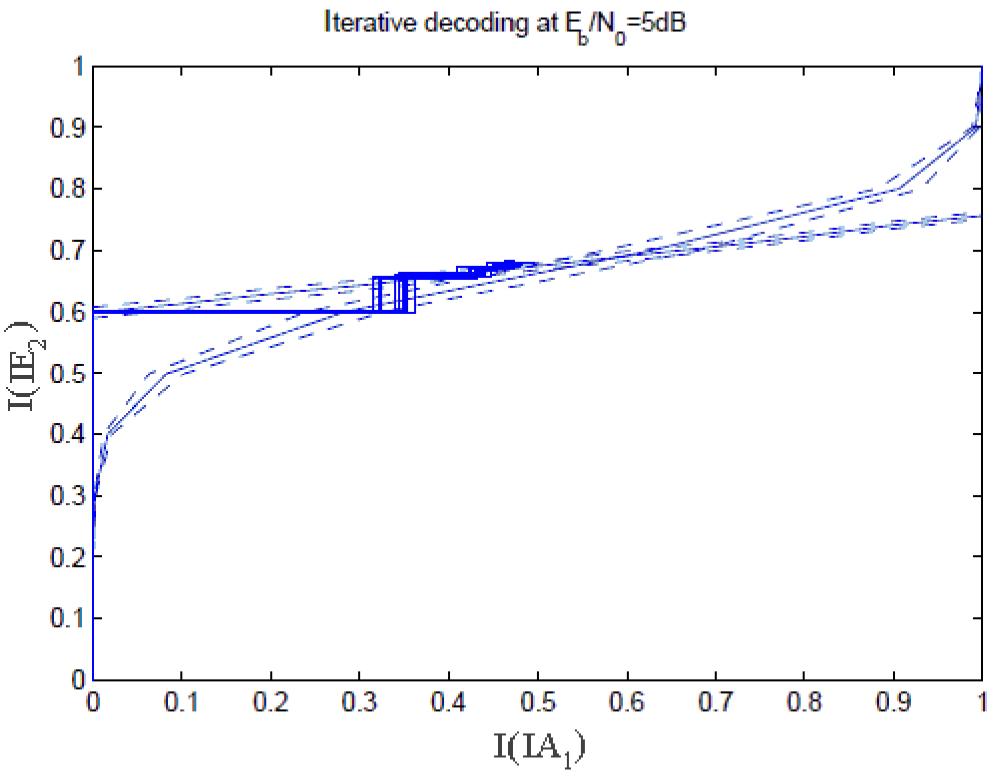


Figure 10.- (Example) Exit chart for iterative decoding when Eb/N0 is 5dB

Chapter 5

5. SIMULATION RESULTS

Having completed the background literature review on the principles of Alamouti scheme, BICM and BICM-ID, this section will evaluate the performance of each of above said for the transmission over AWGN and Rayleigh fading channels. In order to take to end all these simulations, all the functions are been written in Matlab. The simulation carried out in this section will be using a 1/2 rate encoder and different coded modulations for just evaluating which one is the best one for each scheme used. These comparisons are done by using BER curve characteristics. Finally, a case study is been done by using EXtrinsic Information Transfer Charts (EXIT charts) for the modified scheme BI-STCM-ID in order to find the optimal value for .

5.1. Alamouti scheme

In order to be able to see the effect between these of a scheme without diversity in time, that is to say, with a single transmitting antenna and a single receiving antenna, and the use of a scheme with diversity we have realised several simulations.

In order to represent the scheme with diversity we have considered 2 transmitting antennas and the first simulation we are considered with a receiving antenna (2x1) and the second one, with two transmitting antennas(2x2).

Fig. 11 depicts the bit error rate (BER) performance of non-diversity scheme and Alamouti scheme for different number of receiver antennas. It is used BPSK modulation and the information block length is $N = 10^6$.

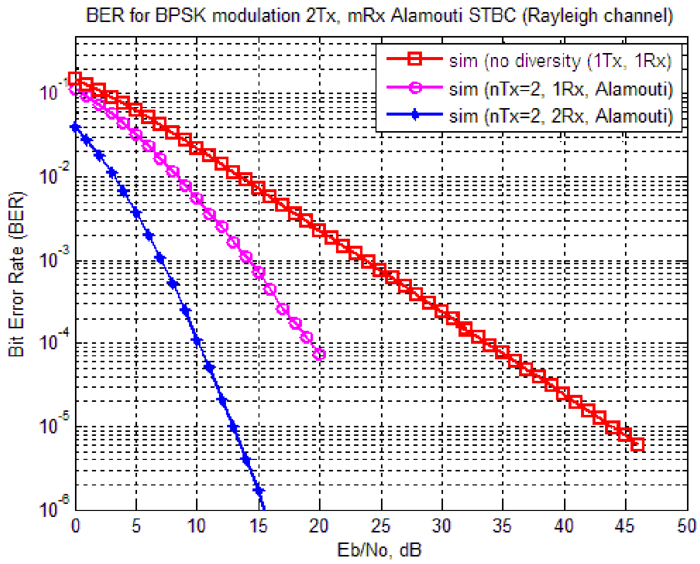


Figure 11.- BER of BPSK two-branch transmit diversity signal using Alamouti Scheme

From this figure, we can observe the improvement that supposes the scheme of Alamouti if it is compared with the system without diversity. In order to obtain an error probability of 10^{-3} bits, it is needed 10dB less for the case a single receiving antenna, and of almost 17 dB less in the Eb/N0 relation for the most complex case of two receiving antennas. So, we can conclude that the diversity gain is really remarkable using space-time diversity (Alamouti scheme) with more than one transmitting antenna.

5.2. Bit –interleaved code modulation (BICM) scheme

In Fig. 12 shows the BER of BICM over AWGN with perfect CSI, where the signal transmitted is a 16QAM coded with 8 states, rate ½ binary convolutional code with (octal) generators (5,7). Then, GRAY and Set-Partitioning labelling (detailed in section 3.2) are compared.

For this model, we can observe that is more suitable Gray labelling than Set-Partitioning labelling. Using Gray labelling it is obtaining an error probability lower than using another constellation mapping. Concretely, to achieve an error probability of 10^{-1} bits, it is needed 2.5dB less for the Gray case.

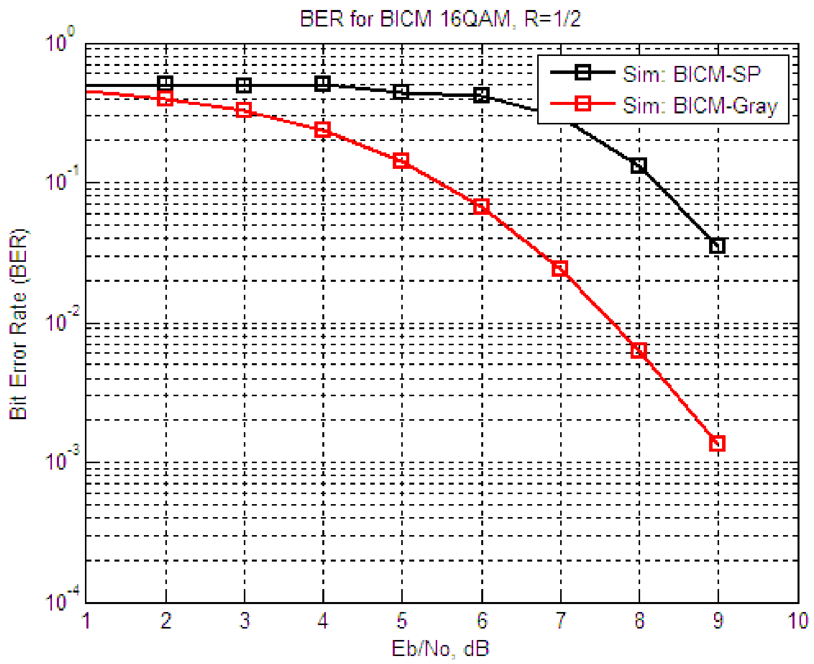


Figure 12.- BER of 16-QAM, 8-states, AWGN channel, rate R=1/2, Gray and Set Partitioning Labelling (SP) using BICM scheme

5.3. BICM-ID scheme

It is shown in [11] that using suitable mappings, the error performance of BICM-ID significantly improves over that of the original BICM with Gray mapping.

Since it has been showed in the previous section, the performance of each model depends on the signal labelling methods. For that reason, besides the constellations defined in section 2.3, it is also defined another 16-QAM constellation using in the next simulations. It is a mixed of Set partitioning Mapping. It is show in [5] that outperforms both Gray labelling and set-partitioning labelling.

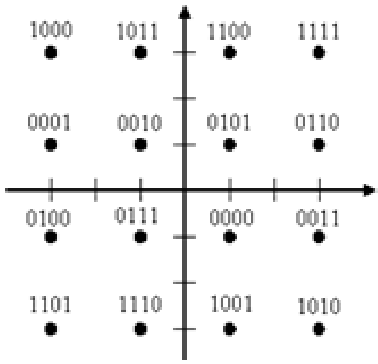


Figure 13.- 16 QAM signal set with Mixed-Set-Partitioning (MSP).

Fig. 14 depicts the performance of BICM-ID with 16-states, rate ½ codes with MSP labelling. With soft decision feedback, it is demonstrated that with each iteration the error probability decreases considerably. As it is explained in section 3.3, this fact is due to the consideration of a priori information (LLR soft-values) as input of the demapping block.

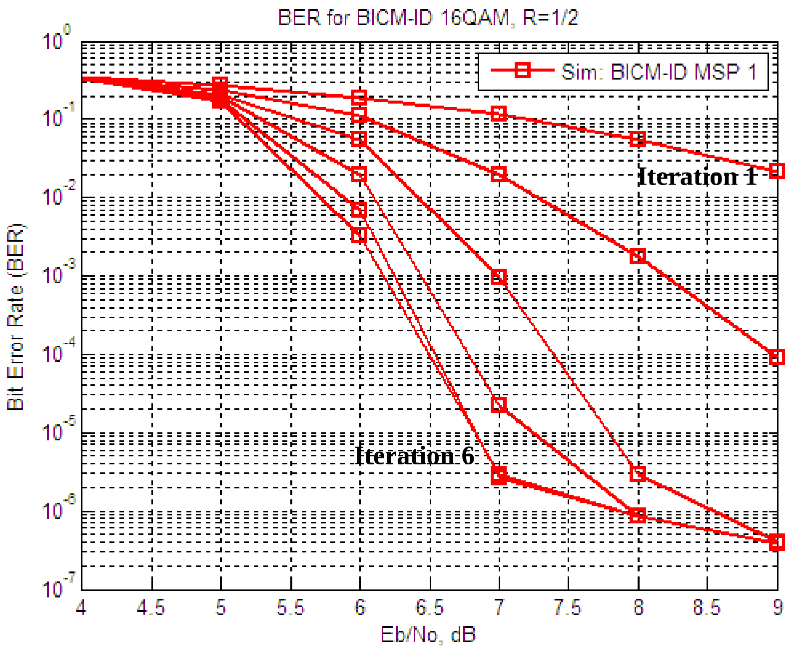


Figure 14.- BER of 16-QAM, 16-states gen = [15 17], Rayleigh channel,2000 bits/block, MSP labelling using BICM-ID scheme

The asymptotic coding gain of about 7dB for MSP labelling can be achieved at BER of 10^{-6}.

For this model, It has been also analyzed the effect that produces the variation, in the simulation, of block length. For this reason, in Fig. 15 it is shown the behaviour of the error probability, in terms of BER, for different values of block length. The considered values are 1000, 2000 and 4000 bits/block.

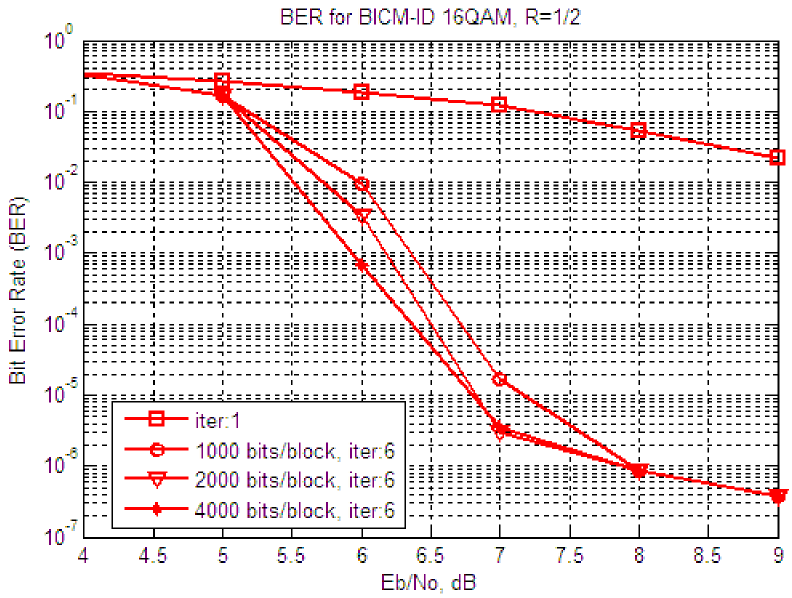


Figure 15.- BER of 16-QAM, 16-states gen = [15 17], Rayleigh channel, MSP labelling, to show the effects of block length on the performance of BICM-ID

It is taking as a reference the previous simulation, in which has been used 2.000 bits/block. Then, when the block length is reduced to half, 1000 bits/block, the loss is about 0.7 dB for BER 10^{-5}. However, when the block length is doubled, 4000 bits/block, there is not a significant improvement in its performance. And, moreover, the time of the simulation increases noticeably. Therefore, using a block length of 2000 bits/block is the best length to achieve the optimal performance.

5.4. BI-STCM/BI-STCM-ID scheme

a. BER:

a.1. Without iterations

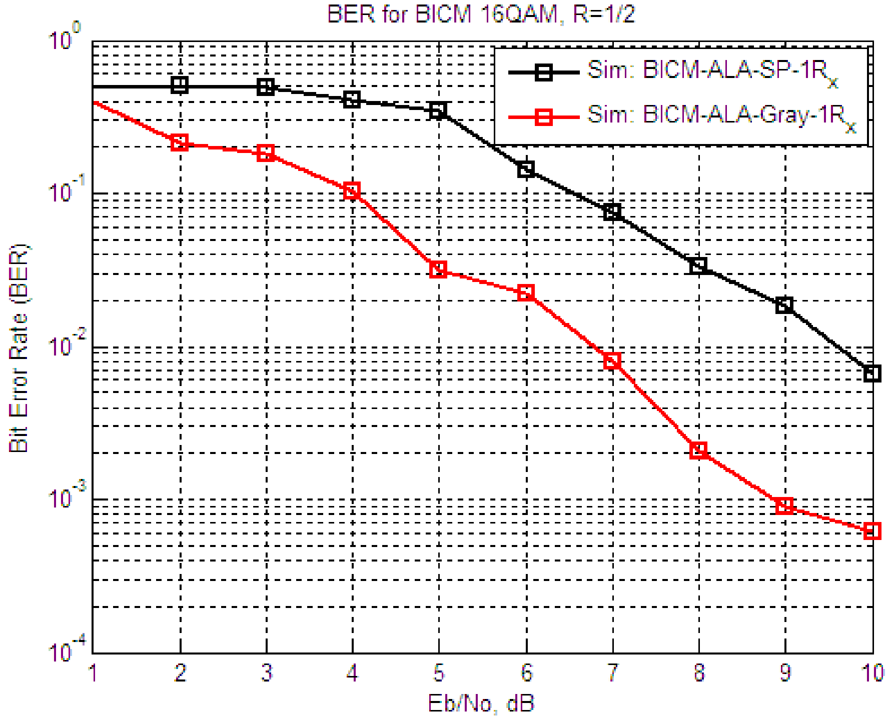


Figure 16.- BER of 16-QAM, 64-states gen = [133 177], AWGN channel, Gray and SP labelling using BI-STCM scheme for 2Tx and 1Rx

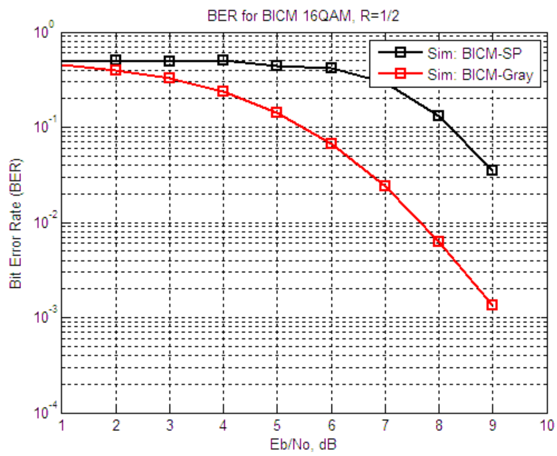


Figure 12.-

a.2. With iterations (BI-STCM-ID - scheme)

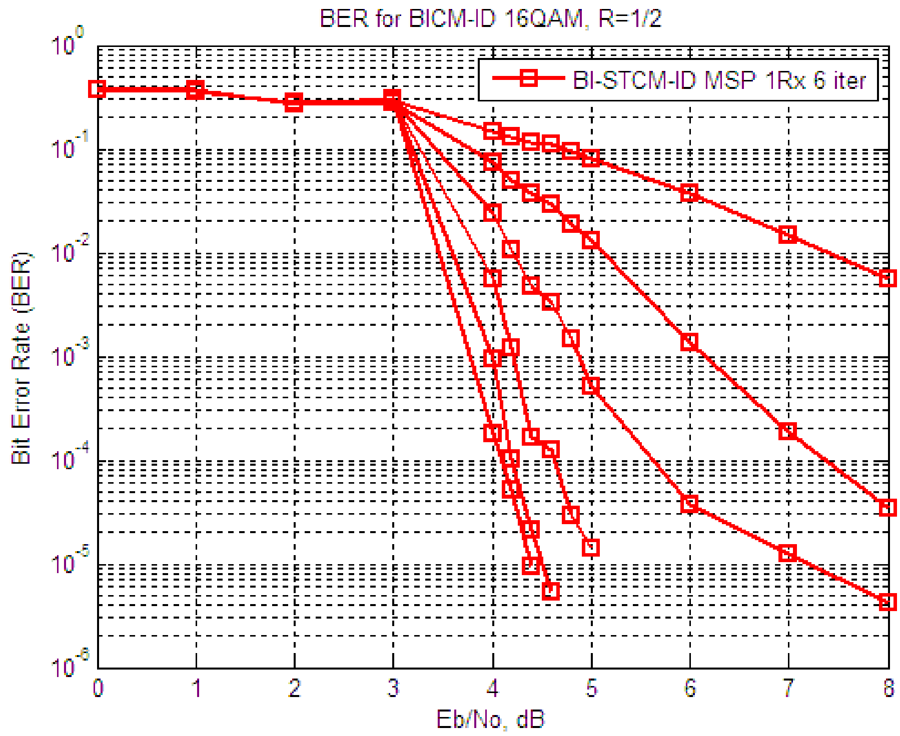


Figure 17.- BER of 16-QAM, 64-states gen = [133 177], AWGN channel, Gray and SP labelling using BI-STCM –ID scheme for 2Tx and 1Rx (6 iterations)

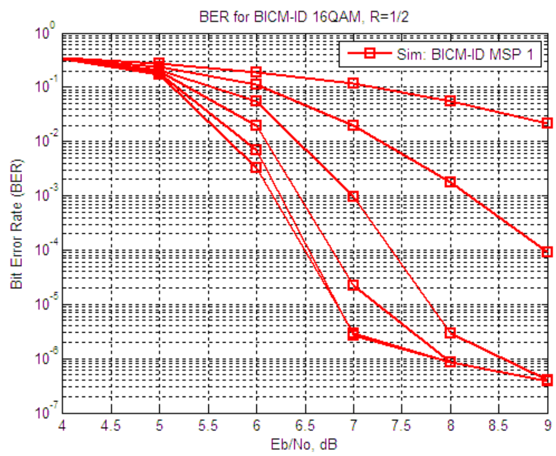


Figure 14.-

b. Exit Chart BI-STMC-ID

In this section it is studied the BI-STMC-ID scheme through EXIT chart, instead of through BER curves characteristics, as until now.

First, it carries out a simulation to show the importance of the block length in this kind of schemes that includes a bit-interleaver and bit-deinterleaved. After that, several simulations are depicted in order to try to obtain the optimal value of β (defined in section 3.5).

The convergence between the demmaper and decoder can be explained easier with the help of the EXIT chart trajectories, as these trajectories will explain how easily the mutual information are passed between each decoder until they converges. The figure 18 and figure 19 shows the comparison of EXIT chart trajectories for bit random interleaver of length of 3000 and 300 at an SNR =4.5dB. The number of iteration required for the convergence of the decoders is only three, which is clearly evident from the Figure 17. Exit graph with widen as well as become narrow according to increase in the frame length.

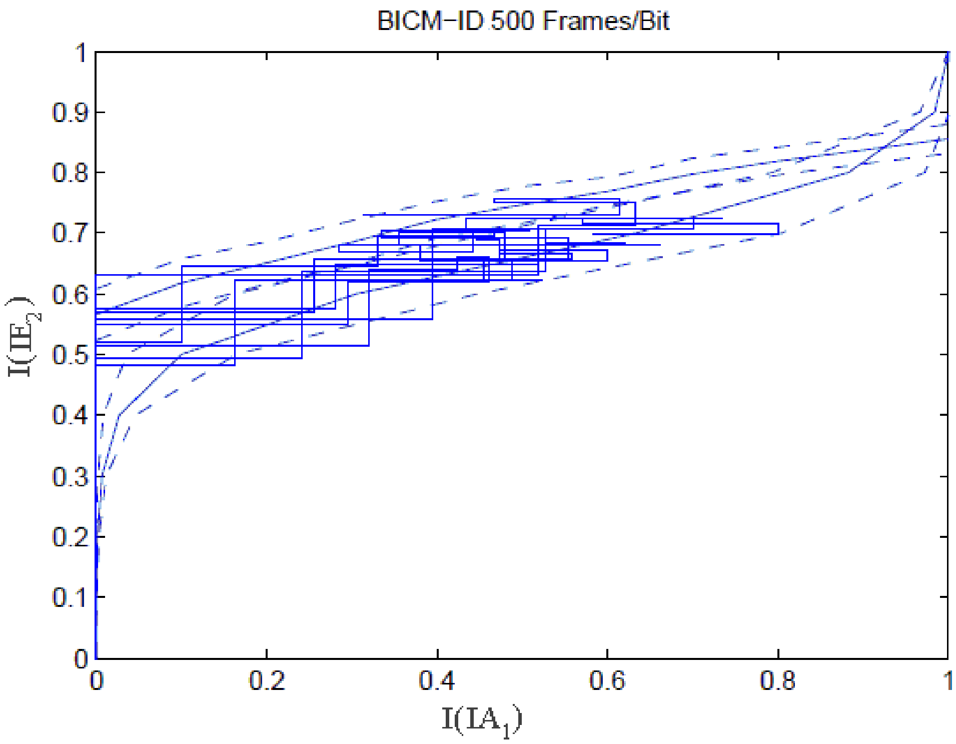


Figure 18.- EXIT Chart for BI-STCM-ID with 500 bits/frame at a SNR =4.5 dB.

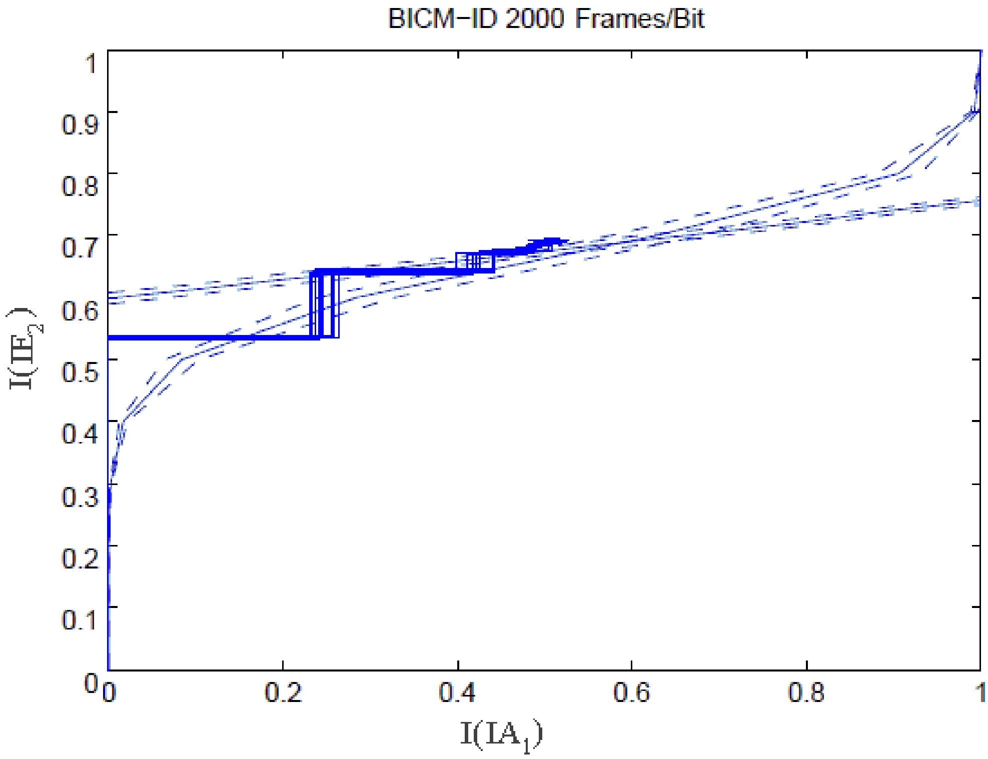


Figure 18.- EXIT Chart for BI-STCM-ID with 2000 bits/frame at a SNR =4.5dB.

Chapter 6

6. CONCLUSIONS

In this project, we present a comprehensive study on the design and analysis of bit-interleaved coded modulation (BICM) with iterative decoding (BICM-ID) for single-antenna and multiple-antenna wireless communications and also a study on the design and analysis of combination of BICM transmitter scheme and space-time codes. The major findings are listed below:

- Alamouti scheme provides space-time diversity. Subsequently, it is shown the improvement that supposes the scheme of Alamouti if it is compared with the system without diversity. So, we can conclude that the diversity gain is really remarkable using space-time diversity (Alamouti scheme) with more than one transmitting antenna.

- For BICM model we can observe that is more suitable Gray labelling than Set-Partitioning labelling. Using Gray labelling it is obtaining an error probability lower than using another constellation mapping.

- Thanks to the iterative iterations in BICM scheme, the performance has a remarkably improvement related to the probability of error.

- BICM-ID performs best when they have a long interleaver length. However, if it is too long, the time of the simulation increases noticeably and the improvement is not really quantifiable. So, it is necessary to find equilibrium between both variables.

- Furthermore, we extend conventional BICM-ID to multiple-input multiple-output (MIMO) channels, and propose bit-interleaved space-time coded modulation (BI-STCM) with iterative decoding (BI-STCM-ID) over Rayleigh fading channels with N_t transmitting and N_r receiving antennas. Numerical results show that the optimal labelling maps achieve significant improvements in the asymptotic coding gain for these constellations. We can conclude that BI-STCM-ID is a really good candidate for the next generation of wireless networks.

APPENDIX

A. Algorithm BCJR decoding proposed by Sarah J. Johnson [7]

B. MATLAB Functions

A. Algorithm BCJR decoding proposed by Sarah J. Johnson [7]

Algorithm 4.2 Log BCJR decoding

```
procedure LOGBCJR(trellis,A,R)
                                                        ▷ Initialization
    for t = 1 : T do
        for r = 0 : 2^ν − 1 do
            for s = 0 : 2^ν − 1 do
                Γ_t(S_r, S_s) = −∞
                if S_r to S_s is a trellis edge then
                    Γ_t(S_r, S_s) = 0
                    for i = 1 : k do
                        if u_{r,s}^(i) = 0 then
                            Γ_t(S_r, S_s) = Γ_t(S_r, S_s) + log( e^{A_t^(i)} / (1 + e^{A_t^(i)}) )
                        else
                            Γ_t(S_r, S_s) = Γ_t(S_r, S_s) + log( e^{−A_t^(i)} / (1 + e^{−A_t^(i)}) )
                        end if
                    end for

                    for i = 1 : n do
                        if c_{r,s}^(i) = 0 then
                            Γ_t(S_r, S_s) = Γ_t(S_r, S_s) + log( e^{R_t^(i)} / (1 + e^{R_t^(i)}) )
                        else
                            Γ_t(S_r, S_s) = Γ_t(S_r, S_s) + log( e^{−R_t^(i)} / (1 + e^{−R_t^(i)}) )
                        end if
                    end for
                end if
            end for
        end for
    end for
    A_t(S_i) = −∞ ∀i, t except
    A_0(S_0) = 0                                        ▷ Forward recursion
    for t = 1 : T − 1 do
        for r = 0 : 2^ν − 1 do
            for s = 0 : 2^ν − 1 do
                A_t(S_s) = max*(A_t(S_s), A_{t−1}(S_r) + Γ_t(S_r, S_s))
            end for
        end for
    end for

    Continued over the page
```

Algorithm 4.2 Log BCJR decoding continued

if terminated **then** ▷ Backward recursion
  $\mathcal{B}_T(S_0) = 0$ and $\mathcal{B}_T(S_{i\neq 1}) = -\infty$
else
  $\mathcal{B}_T(S_i) = \log(1/2^v)$
end if
$\mathcal{B}_t(S_i) = -\infty \; \forall i,\; t < T$
for $t = T - 1 : 1$ **do**
  for $r = 0 : 2^v - 1$ **do**
    for $s = 0 : 2^v - 1$ **do**
      $\mathcal{B}_t(S_r) = \max^*(\mathcal{B}_t(S_r),\; \mathcal{B}_{t+1}(S_s) + \Gamma_{t+1}(S_r, S_s))$
    end for
  end for
end for

for $t = 1 : T$ **do** ▷ State transition probabilities
  for $r = 0 : 2^v - 1$ **do**
    for $s = 0 : 2^v - 1$ **do**
      $M_t(S_r, S_s) = \mathcal{A}_{t-1}(S_r) + \Gamma_t(S_r, S_s) + \mathcal{B}_t(S_s)$
    end for
  end for
end for

for $t = 1 : T$ **do** ▷ Bit probabilities
  for $i = 1 : k$ **do**
    $L_{\text{plus}} = L_{\text{minus}} = -\infty$
    for $r = 0 : 2^v - 1$ **do**
      for $s = 0 : 2^v - 1$ **do**
        if $(S_r, S_s) \in S^{(i)}+$ **then**
          $L_{\text{plus}} = \max^*(L_{\text{plus}},\; M_t(S_r, S_s))$
        else
          $L_{\text{minus}} = \max^*(L_{\text{minus}},\; M_t(S_r, S_s))$
        end if
      end for
    end for
    $L(u_t^{(i)}|\mathrm{y}) = L_{\text{plus}} - L_{\text{minus}}$
  end for
end for
end procedure

B. MATLAB Functions

ALAMOUTI_SCHEME:

```
function BER_simulated = Alamouti(M_Rx)
%ALAMOUTI
%2Tx & M_Rx

global N;          %Simulation samples
global b;          %Random bits
global EbN0dB;     %Eb/N0 values
global s;          %Modulation BPSK
global N_symb;

for i = 1:length (EbN0dB)

   EbN0(i) = 10^(-EbN0dB(i)/20);
   error_counts=0;
   subframes=0;
   bit_count = 0;
   fprintf('Iteracion:!%f \t\t %f \n',i);
   while (error_counts <= 1000 && subframes <=100)

   %Signal block
   signal_cod = zeros(2,N_symb);
   signal_cod(:,1:2:end) = (1/sqrt(2))*reshape(s,2,N_symb/2); %[x1 x2...
   signal_cod(:,2:2:end) = (1/sqrt(2))*(kron(ones(1,N_symb/2),[-
1;1]).*flipud(reshape(conj(s),2,N_symb/2))); %[-x2* x1*...

  % Channel
   h = ones(M_Rx,N_symb); %AWGN
   n = zeros(M_Rx,N_symb);
   %h = 1/sqrt(2)*[randn(M_Rx,N_symb) + j*randn(M_Rx,N_symb)]; %Rayleigh channel
   %n = 1/sqrt(2)*[randn(M_Rx,N_symb) + j*randn(M_Rx,N_symb)]; %Gaussian Noise

   for k = 1:M_Rx
      h_mod = kron(reshape (h(k,:),2,N_symb/2),ones(1,2)); %Fading is constant across two consecutive
symbols

      %Recieved signals
      y_aux = sum(h_mod.*signal_cod,1) + EbN0(i)*n(k,:);
      y([2*k-1,2*k],:) = kron(reshape(y_aux,2,N_symb/2),ones(1,2)); %Señal Para el combinador

      %Modificación de la señal del canal conocida por el receptor
      h_comb([2*k-1,2*k],:) = h_mod;
      h_comb(2*k-1,[2:2:end]) = h_comb(2*k,[1:2:end]);
      h_comb(2*k,[2:2:end]) = -h_comb(2*k-1,[1:2:end]);
   end

   y([2:2:end],:) = conj(y([2:2:end],:));
   h_comb([1:2:end],:) = conj(h_comb([1:2:end],:));

   %The combining Scheme
   pot_hcomb = sum(h_comb.*conj(h_comb),1);
   signal_comb = sum(h_comb.*y,1)./pot_hcomb;

   %The maximum Likelihood Decisor Rule
   b_est = real(signal_comb)>0; %Decisor para BPSK
   subframes=subframes+1;
```

```
    total_error = size(find([b-b_est]),2);
    error_counts = error_counts + total_error;
    bit_count = bit_count+sum(b ~= b_est)+sum(b == b_est);

    end
    %Error counter
    n_err(i) = error_counts/bit_count;
    if n_err(i) == 0;
        while i+1<length (EbN0dB)
            n_err(i)=0;
            i=i+1;
        end
        break;
    end
end
BER_simulated = n_err; %Ber simulada.
end
```

BICM_SCHEME:

```
function [Ber_simulated,error_counts,bit_count] = BICM
(mode_in,k,n,L,generator,tail,bits_per_symbol,mode,modulation,bits_per_frame,EbN0dB,iterations)

global intrlv_param BICM_param mod_param cod_param
%%%%%%%%%%%%%%% Initialize encoder parameters
%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    cod_param.gen=generator;
    cod_param.tail=tail;

%%%%%%%%%%%%%%% Initialize interleaver parameters
%%%%%%%%%%%%%%%%%%%%%%%%

    intrlv_param=struct([]);
        intrlv_param(1).length = bits_per_frame;     % interleaver length
        intrlv_param.prueba=0;
%%%%%%%%%%%%%% Initialize BICM parameters
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    BICM_param=struct([]);
        BICM_param(1).k = k;
        BICM_param.n = n;
        BICM_param.L = L;
        BICM_param.bits_per_symbol = bits_per_symbol;
        BICM_param.bits_per_frame_in = bits_per_frame;
        BICM_param.bits_per_frame_in_t = bits_per_frame+L;
        BICM_param.no_of_coded_bits = bits_per_frame*n/k;
        BICM_param.no_of_coded_bits_t = (bits_per_frame+L)*n/k;
        BICM_param.iterations=iterations;
        BICM_param.mode_in=mode_in;

%%%%%%%%%%%%%%%%%%%% Initialize modulation parameters
%%%%%%%%%%%%%%%%%%%%

    mod_param = struct([]);
        mod_param(1).mode = mode;
        mod_param.mod=modulation;
        mod_param.bits_per_symbol = bits_per_symbol;
```

```
        mod_param.number_of_levels = bitshift(1,bits_per_symbol);
        mod_param.SymbolMapping = zeros((mod_param.number_of_levels),1);
        mod_param.no_of_symbols = bits_per_frame*n/bits_per_symbol;
        mod_param.no_of_symbols_t = (bits_per_frame+L)*n/bits_per_symbol;

        switch(BICM_param.mode_in)
        case {'BICM'}
          errorlimit= repmat(2000,1,length(EbN0dB));
          switch(mod_param.mod)
              case{'SP'}
              Modulation_SP();  % BICM-ID
              fprintf('Modulation_SP\n');
              case{'MSP'}
              Modulation_MSP();
              fprintf('Modulation_MSP\n');
              case {'GRAY'}
              Modulation_Gray();
              fprintf('Modulation_GRAY\n');
              BICM_param.iterations=1;
           end
        case {'BICM-ID'}
          errorlimit=[2000 2000 2000 2000 2000 2000 500 100 100 100 100 50 50 50];
          switch(mod_param.mod)
             case{'SP'}
              Modulation_SP();  % BICM-ID
              fprintf('Modulation_SP\n');
             case{'MSP'}
              Modulation_MSP();
              fprintf('Modulation_MSP\n');
              case {'GRAY'}
              Modulation_Gray();
              fprintf('Modulation_GRAY\n');
          end
        end
    fprintf('Mode: \t %s \nBits per symbol: \t %f \nNumber of levels \t %f
\n',mode,bits_per_symbol,mod_param.number_of_levels);
    fprintf('Iterations: \t %f \n',iterations);
 %%%%%%%%%%%%%%%%%%%%%%%%%

Es = 1;       %Energy per symbol
no_of_info_bits = bits_per_frame;
no_of_coded_bits = bits_per_frame*n/k;
ber = zeros(iterations,length(EbN0dB));


EbN0=zeros(1,length(EbN0dB));
R=(no_of_info_bits/no_of_coded_bits); %Code rate
Eb=Es/(R*mod_param.bits_per_symbol);  %Energy per bit

for i=1:length(EbN0dB)

        bit_count=0;
        EbN0(i) = 10^(EbN0dB(i)/10);
        subframes=0;
        fprintf('Iteracion:!%f \t\t %f \n',i);
        error_counts = zeros(iterations,1);
        N0 = Eb/(EbN0(i));     %Sigma

        while (error_counts(iterations,1) <= errorlimit(1,EbN0dB(i)))
```

```
        %load('prueba3.mat')
        b_Input_bits = round(rand(1,no_of_info_bits));  %Input bits
        %b_Input_bits(1,[1:12])
        % Transmitted signal
        BICM_param.b_Input_bits_iter = kron(b_Input_bits,ones(iterations,1));

        TX_Signal = transmitter(b_Input_bits);   % encode, interleaver & modulate

       %%% Channel
        sizeTX=size(TX_Signal);

       %AWGN
       %channel = ones(sizeTX);
       %Rayleigh fading channel
       channel = 1/sqrt(2)*[randn(sizeTX) + 1i*randn(sizeTX)];

       %n = zeros(size(TX_Signal));
        n = sqrt(N0/2)*[randn(sizeTX)+1i*randn(sizeTX)];

        %Received signal


        RX_Signal =TX_Signal.*channel + n;       % y = x*h + n


        b_decoded_bits = receiver(RX_Signal,N0,channel);   % demodulate, deinterleave & decode

        b_Input_bits_iter = kron(b_Input_bits,ones(iterations,1));
        total_error = sum([b_Input_bits_iter ~= b_decoded_bits(:,1:bits_per_frame)],2);

        error_counts = error_counts+total_error;
        bit_count = bit_count + sum(b_decoded_bits(:,1:bits_per_frame)~=
b_Input_bits_iter,2)+sum(b_decoded_bits(:,1:bits_per_frame) == b_Input_bits_iter,2);
        subframes = subframes+1

        %Pause-Control
        pausecmd = dir('sometoken');
        if pausecmd.bytes > 1
          delete('sometoken')
          fclose(fopen('sometoken','w'));
          keyboard
        end
      end
   ber(:,i) = error_counts./bit_count;
   if error_counts==0
      break;
   end
end
Ber_simulated = ber;
end
```

RECEIVER_BICM/BICM-ID

```
function [decoded_bits] = receiver(RX_Signal,N0,channel)

    global BICM_param cod_param
    BICM_param.iteration_num = 1;

    T = BICM_param.bits_per_frame_in;
    Apr = zeros(1,T);
    Apr(:,:) = log(0.5/0.5);
    Apr_codw = zeros(1,BICM_param.no_of_coded_bits);

    Dist_matrix = soft_demodulate(RX_Signal,N0,channel); % p(y|s)

    switch(BICM_param.mode_in)
        case {'BICM'}  %% max-log-likelihood bit metric//VITERBI decoder
            R = LLR (Dist_matrix,Apr_codw);
            R_deintrv = deinterleaver(R);
            decoded_bits = vitdec(R_deintrv,cod_param.trellis,12,'trunc','hard');

        case {'BICM-ID'}
            Ext2 = Apr_codw;
            for l = 1 : BICM_param.iterations
                Apr1 = rand_interleaver(Ext2(l,:),0);
                L1 = LLR(Dist_matrix,Apr1);
                Apr2(l,:) = deinterleaver(L1);
                [L2infoword(l,:),L2codeword(l,:)]= BCJR_log(cod_param.trellis,Apr,Apr2(l,:));
                Ext2(l+1,:) = L2codeword(l,:)- Apr2(l,:);
            end
            decoded_bits = hard_decision(L2infoword);
            decoded_cobits = hard_decision(L2codeword);

            total_error = sum(BICM_param.b_Input_bits_iter ~= decoded_bits,2);
        otherwise
             error('Error mode_in (BICM or BICM-ID)');
     end
end
```

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